Heavy-light $\bar{Q}q$ mesons in QCD [*]

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This talk summarizes the study of the dynamics of the heavy-light $\bar{Q}q$ open charm and beauty mesons obtained in [1] using QCD spectral sum rules (QSSR) and motivated by the recent experimental discovery of the $D_{sJ}(2317)$ and $D_{sJ}(2457)$ mesons. The important rôle of the chiral condensate $\langle\bar{\psi}\psi\rangle$ in the mass-splittings between the scalar-pseudoscalar mesons is emphasized. The emerging value of the running charm quark mass for reproducing the well-known $D(0^-)$ and $D_s(0^-)$ masses is: $\bar{m}_c(m_c) = 1.13^{+0.08}_{-0.04}$ GeV, which confirms previous estimates from this channel [2]. Using this value, the sum rules give: $M_{D_s(0^+)} \simeq (2297 \pm 113)$ MeV, and a small $SU(3)$ breaking: $M_{D_s(0^+)} - M_{D(0^+)} \approx 25$ MeV. Extending the analysis to the B-system, one finds: $M_{B(0^+)} - M_{B(0^-)} \simeq (422 \pm 196)$ MeV $\simeq M_{D_s(0^+)} - M_{D_s(0^-)}$. Assuming an approximate (heavy and light) flavour and spin symmetries of the mass-splittings as indicated by the previous results, one also deduces: $M_{D_s^*(1^+)} \simeq (2440 \pm 113)$ MeV. Finally, one also gets: $f_{D(0^+)} \simeq (217 \pm 25)$ MeV much bigger than $f_\pi = 130.6$ MeV, suggesting a large violation of the $1/\sqrt{M_D}$ scaling, while the size of the $SU(3)$ breaking ratio $f_{D_s(0^+)}/f_{D(0^+)} \simeq 0.93 \pm 0.02$ is opposite to the one of the 0^- channel of about 1.14.

1. INTRODUCTION

The recent observations of two new states $D_{sJ}(2317)$ and $D_{sJ}(2457)$ [3] in the $D_s\pi$, $D_s\gamma$ and $D_s\pi\gamma$ final states have stimulated a renewed interest in the spectroscopy of open charm states which one can notice from different recent theoretical attempts to identify their nature [4]. In a recent paper [1], we have tried to provide the answer to this question from QCD spectral sum rules à la Shifman-Vainshtein-Zakharov [5]. In fact, a similar question has been already addressed in the past [6], where we have predicted using QSSR the mass splitting of the $0^+ - 0^-$ and $1^- - 1^+$ $\bar{b}u$ mesons using double ratio of moments sum rules based on an expansion in the inverse of the b quark mass. We found that the value of the mass-splittings between the chiral multiplets were about the same and approximately independent of the spin of these mesons signaling an heavy quark-type approximate symmetry:

$$
\begin{aligned}
M_{B(0^+)} - M_{B(0^-)} &\approx M_{B^*(1^+)} - M_{B^*(1^-)} \\
&\approx (417 \pm 212) \text{ MeV} .
\end{aligned} \tag{1}
$$

The effect and errors on the mass-splittings are mainly due to the chiral condensate $\langle\bar{\psi}\psi\rangle$ and to the value of the b quark mass. In the paper [1], we have used an analogous approach to the open charm states. However, a method in terms of the $1/m_c$ expansion and some other nonrelativistic sum rules will be dangerous here due to the relatively light value of the charm quark mass. Instead, we shall work with

relativistic exponential sum rules used successfully in the light quark channels for predicting the meson masses and QCD parameters [7] and in the D and B channels for predicting the (famous) decay constants $f_{D,B}$ [2,7,8] and the charm and bottom quark masses [2,7,8,9,10].

2. THE SUM RULES

We shall work here with the (pseudo)scalar two-point correlators:

$$
\psi_{P/S}(q^2) \equiv i \int d^4x \, e^{iqx} \, \langle 0|\mathcal{T} J_{P/S}(x) J^\dagger_{P/S}(0)|0\rangle, \tag{2}
$$

built from the (pseudo)scalar and (axial)-vector heavy-light quark currents:

$$
J_{P/S}(x) = (m_Q \pm m_q)\bar{Q}(i\gamma_5)q, \; J^\mu_{V/A} = \bar{Q}\gamma^\mu(\gamma_5)q . \tag{3}
$$

If we fix $Q \equiv c$ and $q \equiv s$, the corresponding mesons have the quantum numbers of the $D_s(0^-)$, $D_s(0^+)$ mesons. m_Q and m_s are the running quark masses. In the (pseudo)scalar channels, the relevant sum rules for our problem are the Laplace transform sum rules:

$$
\begin{aligned}
\mathcal{L}^H_{P/S}(\tau) &= \int_{t_\leq}^\infty dt \, e^{-t\tau} \, \frac{1}{\pi} \, \text{Im}\psi^H_{P/S}(t), \\
\mathcal{R}^H_{P/S}(\tau) &\equiv -\frac{d}{d\tau} \log \mathcal{L}^H_{P/S}(\tau),
\end{aligned} \tag{4}
$$

where $t_\leq$ is the hadronic threshold, and H denotes the corresponding meson. The latter sum rule, or its slight modification, is useful, as it is equal to the resonance mass squared, in the simple duality ansatz parametrization of the spectral function:

$$
\begin{aligned}
\frac{1}{\pi} \text{Im}\psi^H_P(t) &\simeq f_D^2 M_D^4 \delta(t - M_D^2) \\
&+ \text{``QCD continuum''} \Theta(t - t_c), \tag{5}
\end{aligned}
$$

where the "QCD continuum comes from the discontinuity of the QCD diagrams, which is expected to give a good smearing of the different radial excitations [2]. The decay constant f_D is analogous to $f_\pi = 130.6$ MeV; t_c is the QCD continuum threshold, which is, like the sum rule variable τ, an (a priori) arbitrary parameter. In this paper, we shall impose the τ- and t_c-stability criteria for extracting our optimal results. The corresponding t_c value also agrees with the FESR duality constraints [11,7] and very roughly indicates the position of the next radial excitations. However, in order to have a conservative result, we take a largest range of t_c from the beginning of τ- to the one of t_c-stabilities.

The QCD expression of the correlator is well-known to two-loop accuracy (see e.g. [7] and the explicit expressions given in [2,8]), in terms of the perturbative pole mass M_Q, and including the non-perturbative condensates of dimensions less than or equal to six [3]. For a pedagocial presentation, we write the sum rule in the chiral limit ($m_s = 0$) and to leading order in α_s, where the expression is more compact. In this way, one can understand qualitatively the source of the mass splittings. The sum rule reads to leading order:

$$
\mathcal{L}_{P/S}^H(\tau) = M_Q^2 \Bigg\{ \int_{M_Q^2}^{\infty} dt\ e^{-t\tau} \frac{1}{8\pi^2} 3t(1-x)^2
$$
$$
+ \ C_4\langle O_4\rangle_{P/S} + \tau C_6\langle O_6\rangle_{P/S}\ e^{-M_Q^2\tau} \Bigg\}(6)
$$

where $\langle O_{4(6)}\rangle$ are the dimension-4(6) condensates and and $C_{4(6)}$ their respective Wilson coefficients:

$$
x \equiv M_Q^2/t,
$$
$$
C_4\langle O_4\rangle_{P/S} = \mp M_Q\langle \bar{d}d\rangle\ e^{-M_Q^2\tau}
$$
$$
+\langle \alpha_s G^2\rangle \left(\frac{3}{2} - M_Q^2\tau\right)/12\pi,
$$
$$
C_6\langle O_6\rangle_{P/S} = \mp \frac{M_Q}{2}\left(1 - \frac{M_Q^2\tau}{2}\right) \times
$$
$$
g\langle \bar{d}\sigma_{\mu\nu}\frac{\lambda_a}{2}G_a^{\mu\nu}d\rangle
$$
$$
-\left(\frac{8\pi}{27}\right)\left(2 - \frac{M_Q^2\tau}{2} - \frac{M_Q^4\tau^2}{6}\right) \times
$$
$$
\rho\alpha_s\langle \bar{\psi}\psi\rangle^2,
\tag{7}
$$

where we have used the contribution of the gluon condensate given in [13], which is IR finite when letting $m_q \to 0$ [4]. The previous sum rules can be ex-

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[2]At the optimization scale, the continuum effect is negligible, such that a more involved parametrization is not necessary.

[3]We shall include the negligible contribution from the dimension six four-quark condensates, while we shall neglect an eventual tachyonic gluon mass correction term found to be negligible in some other channels [12].

[4]The numerical change is negligible compared with the original expression obtained in [14].

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pressed in terms of the running mass $\bar{m}_Q(\nu)$ through the perturbative two-loop relation [15,16]:

$$
M_Q = \bar{m}_Q(p^2)\left[1 + \left(\frac{4}{3} + \ln\frac{p^2}{M_Q^2}\right)\left(\frac{\bar{\alpha}_s}{\pi}\right)\right], \quad (8)
$$

where M_Q is the pole mass. Throughout this paper we shall use the values of the QCD parameters given in Table 1.

Table 1
QCD input parameters used in the analysis.

Parameters	References
$\Lambda_4 = (325 \pm 43)$ MeV	[7]
$\Lambda_5 = (225 \pm 30)$ MeV	[7]
$\bar{m}_b(m_b) = (4.24 \pm 0.06)$ GeV	[7,10,2]
$\bar{m}_s(2\text{ GeV}) = (111 \pm 22)$ MeV	[7,10,17,18]
$\langle \bar{d}d\rangle^{1/3}(2\text{ GeV}) = -(243 \pm 14)$ MeV	[7,10,19]
$\langle \bar{s}s\rangle/\langle \bar{d}d\rangle = 0.8 \pm 0.1$	[7,20]
$\langle \alpha_s G^2\rangle = (0.07 \pm 0.01)$ GeV^4	[7,21]
$M_0^2 = (0.8 \pm 0.1)$ GeV^2	[7,6]
$\alpha_s\langle \bar{\psi}\psi\rangle^2 = (5.8 \pm 2.4) \times 10^{-4}$ GeV^6	[7,21,22]

We have used for the mixed condensate the parametrization: $g\langle \bar{d}\sigma_{\mu\nu}\frac{\lambda_a}{2}G_a^{\mu\nu}d\rangle = M_0^2\langle \bar{d}d\rangle$, and deduced the value of the QCD scale Λ from the value of $\alpha_s(M_Z) = (0.1184 \pm 0.031)$ [23,24]. We have taken the mean value of m_s from recent papers and reviews [7,10,17,18].

3. $\bar{m}_c(m_c)$ FROM $M_{D(0^-)}$ AND $M_{D_s(0^-)}$

This analysis has been already done in previous papers to order α_s and α_s^2 [2,8] and has served to fix the running charm quark mass. We repeat this analysis here to order α_s for a pedagogical purpose. We show in Fig. 1a), the τ-dependence of the $D(0^-)$ and in Fig 1b) the one of the $D_s(0^-)$ masses for a given value of t_c, which is the central value of the range:

$$
t_c = (7.5 \pm 1.5)\text{ GeV}^2, \tag{9}
$$

where the lowest value corresponds to the beginning of τ-stablity and the highest one to the beginning of t_c-stability obtained by [2,8,7] in the analysis of f_D and f_{D_s}. This range of t_c-values covers the different choices of t_c used in the sum rule literature. As mentioned previously, the one of the beginning of t_c stability coïncides, in general, with the value obtained from FESR local duality constraints [11,7]. Using the input values of QCD parameters in Table 1, the best fits of the $D(0^-)$ (resp. $D_s(0^-)$) masses
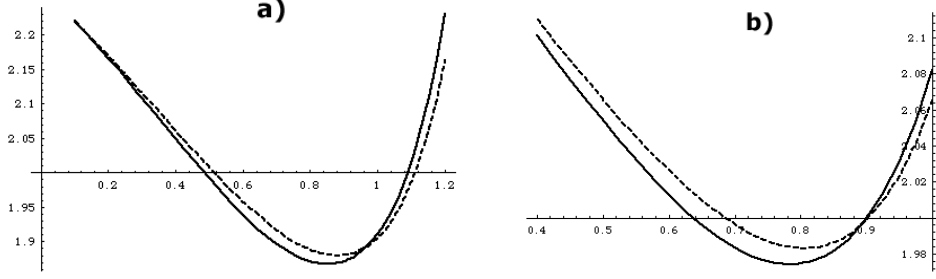
Figure 1. τ in GeV^{-2}-dependence of the a) $M_D(0^-)$ in GeV for $\bar{m}_c(m_c) = 1.11$ GeV and b) $M_{D_s(0^-)}$ in GeV for $\bar{m}_c(m_c) = 1.15$ GeV at a given value of $t_c = 7.5$ GeV^2. The dashed line is the result including the leading $\langle\bar{\psi}\psi\rangle$ contribution. The full line is the one including non-perturbative effects up to dimension-six.

for a given value of $t_c = 7.5$ GeV^2 correspond to a value of $\bar{m}_c(m_c)$ of 1.11 (resp. 1.15) GeV. Taking the mean value as an estimate, one can deduce:

$$\bar{m}_c(m_c^2) = (1.13^{+0.07}_{-0.02} \pm 0.02 \pm 0.02 \pm 0.02) \text{ GeV} , \quad (10)$$

where the errors come respectively from t_c, $\langle\bar{\psi}\psi\rangle$, Λ and the mean value of m_c required from fitting the $D(0^-)$ and $D_s(0^-)$ masses. This value is perfectly consistent with the one obtained in [2,8] obtained to the same order and to order α_s^2, indicating that, though the α_s^2 corrections are both large in the two-point function and m_c [25], it does not affect much the final result from sum rule analysis. In fact, higher corrections tend mainly to shift the position of the stability regions but affect slightly the output value of m_c. This value of m_c is in the range of the current average value (1.23 ± 0.05) GeV reviewed in [7,10,23]. However, it does not favour higher values of m_c allowed in some other channels and by some non relativistic sum rules and approaches. However, these non relativistic approaches might be quite in-accurate due to the relative smallness of the charm quark mass. Higher values of m_c would lead to an overestimate of the $D(0^-)$ and $D_s(0^-)$ masses. In the following analysis, we shall use the central value $\bar{m}_c(m_c) = 1.11$ (resp. 1.15) GeV for the non-strange (resp. strange) meson channels.

4. THE 0^+ MESON MASSES

• We study in Fig. 2), the τ-dependence of the $D_s(0^+)$ mass at the values of t_c and m_c obtained previously. In this way, we obtain:

$$M_{D_s(0^+)} \simeq (2297^{+81+63}_{-98-70} \pm 11 \pm 2 \pm 11) \text{ MeV} \quad (11)$$

where the errors come respectively from t_c, m_c, $\langle\bar{\psi}\psi\rangle$, m_s, and Λ. This implies:

$$M_{D_s(0^+)} - M_{D_s(0^-)} = (328 \pm 113) \text{ MeV} , \quad (12)$$

We have used the experimental value of $M_{D_s(0^-)}$. The reduction of the theoretical error needs precise
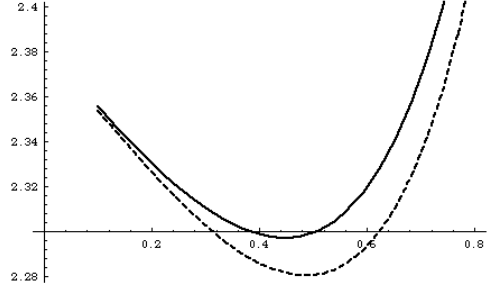

Figure 2. Similar to Fig. 1 but τ-behaviour of $M_{D_s(0^+)}$ for given values of $t_c = 7.5$ GeV^2 and $m_c(m_c) = 1.15$ GeV.

values of the continuum threshold [5] and of the charm quark mass which are not within the present reach of the estimate of these quantities [6] . Further discoveries of the continuum states will reduce the present error in the splitting. One should also notice that in the ratio of sum rules with which we are working, we expect that perturbative radiative corrections are minimized though individually large in the expression of the correlator and of the quark mass.

• The value of the mass-splittings obtained previously is comparable with the one of the $B(0^+)$-$B(0^-)$ given in Eq. (1), and suggests an approximate heavy-

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[5]The range of t_c-values 6-9 GeV^2 obtained previously for the $D(0^-)$ mesons coïncides a posteriori with the corresponding range for the $D(0^+)$ meson if one assumes that the splitting between the radial excitations is the same as the one between the ground states, i.e about 300 MeV. We have cheked during the analysis that this effect is unimportant and is inside the large error induced by the range of t_c used.

[6]For this reason, as explicitly discussed in [1], the error of 30 MeV quoted in [26] has been underestimated. Indeed, it only takes into account the one from a small range of the continuum threshold values.

flavour symmetry of this observable.

- We also derive the result in the limit of $SU(3)_F$ symmetry where the strange quark mass is put to zero, and where the $\langle \bar{\psi}\psi \rangle$ condensate is chirally symmetric ($\langle \bar{s}s \rangle = \langle \bar{d}d \rangle$). In this case, one can predict an approximate degenerate mass within the errors:

$$M_{D_s(0^+)} - M_{D(0^+)} \simeq 25 \text{ MeV} , \qquad (13)$$

which indicates that the mass-splitting between the strange and non-strange 0^+ open charm mesons is almost not affected by $SU(3)$ breakings, contrary to the case of the 0^- mesons with a splitting of about 100 MeV.

- We extend the analysis to the case of the $B(0^+)$ meson. Here, it is more informative to predict the ratio of the 0^+ over the 0^- masses as the prediction on the absolute values though presenting stability in τ tend to overestimate the value of M_B. We obtain:

$$\frac{M_{B(0^+)}}{M_{B(0^-)}} \simeq 1.08 \pm 0.03 \pm 0.03 \pm 0.02 \pm 0.02 \qquad (14)$$

where the errors come respectively from t_c taken in the range $43 - 60 \text{ GeV}^2$, m_b, $\langle \bar{\psi}\psi \rangle$, and τ. We have used the value of $\bar{m}_b(m_b)$ given in Table 1. This implies:

$$M_{B(0^+)} - M_{B(0^-)} \simeq (422 \pm 196) \text{ MeV} , \qquad (15)$$

which agrees with the result in Eq. (1) obtained from moment sum rules [6].

5. The 1^+ MESON MASS

Our previous results in Eqs. (1), (11) to (15) suggest that the mass-splittings are approximately (heavy and light) flavour and spin independent. Therefore, one can write to a good approximation the empirical relation:

$$
\begin{aligned}
M_{D_s(0^+)} - M_{D_s(0^-)} &\approx & M_{D(0^+)} - M_{D(0^-)} \approx \\
M_{B(0^+)} - M_{B(0^-)} &\approx & M_{B(1^+)} - M_{B(1^-)} \\
&\approx & M_{D_s^*(1^+)} - M_{D_s^*(1^-)} . \quad (16)
\end{aligned}
$$

Using the most precise number given in Eq. (11), one can deduce:

$$M_{D_s^*(1^+)} \simeq (2440 \pm 113) \text{ MeV} . \qquad (17)$$

This result is consistent with the 1^+ assignement of the $\bar{c}s$ meson $D_{sJ}(2457)$ discovered recently [3].

6. THE 0^+ DECAY CONSTANTS

For completing our analysis, we estimate the decay constant $f_{D(0^+)}$ analogue to $f_\pi = 130.6$ MeV. We show the behaviour of $f_{D(0^+)}$ versus τ, where a goood stablity is obtained. Adopting the range of t_c-values obtained previously and using $\bar{m}_c(m_c) = 1.11^{+0.08}_{-0.04}$ GeV required for a best fit of the non strange $D(0^+)$ meson mass, we deduce to two-loop accuracy:

$$f_{D(0^+)} = (217^{+5+15}_{-15-19} \pm 10 \pm 10) \text{ MeV} , \qquad (18)$$
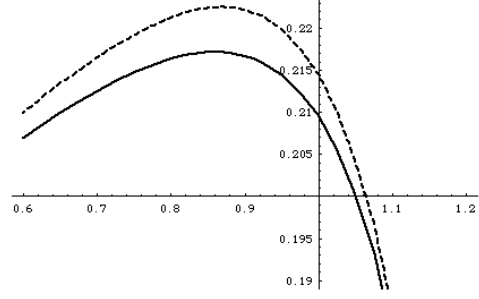

Figure 3. Similar to Fig. 1 but τ-behaviour of $f_D(0^+)$ for given values of $t_c = 7.5 \text{ GeV}^2$ and $\bar{m}_c(m_c) = 1.11$ GeV.

where the errors come respectively from the values of t_c, m_c, $\langle \bar{\psi}\psi \rangle$ and Λ. We have fixed $M_{D(0+)}$ to be about 2272 MeV from our previous fit. It is informative to compare this result with the one of $f_D = (205 \pm 20)$ MeV, where the main difference can be attributed by the sign flip of the quark condensate contribution in the QCD expression of the corresponding correlators. A numerical study of the $SU(3)$ breaking effect leads to:

$$r_s \equiv \frac{f_{D_s(0^+)}}{f_{D(0^+)}} \simeq 0.93 \pm 0.02 , \qquad (19)$$

which is reverse to the analogous ratio in the pseudoscalar channel $f_{D_s}/f_D \simeq 1.14 \pm 0.04$ given semi-analytically in [27]. In order to understand this result, we give a semi-analytic parametrization of this $SU(3)$ breaking ratio. Keeping the leading term in m_s and $\langle \bar{\psi}\psi \rangle$, one obtains:

$$
\begin{aligned}
r_s &\simeq & \left(1 - \frac{m_s}{m_c} \right) \left[1 - 7.5 \langle \bar{s}s - \bar{d}d \rangle \right]^{1/2} \times \\
& & \left(\frac{M_{D_s(0^+)}}{M_{D(0^+)}} \right)^2 \simeq 0.9 , \qquad (20)
\end{aligned}
$$

where the main effect comes from the negative sign of the m_s contribution in the overall normalization of the scalar current, while the meson mass ratio does not compensate this effect because of the almost equal mass of $D_s(0^+)$ and $D(0^+)$ obtained in previous analysis. This feature is opposite to the case of $f_{D(0^-)}$.

7. CONCLUSIONS

Motivated by the experimental recent discovery of the $D_{sJ}(2317)$ and $D_{sJ}^*(2457)$, we have analyzed in [1] using QSSR the dynamics of the $0^\pm$ and $1^\pm$ open charm and beauty meson channels. Then, we have:

- Re-estimated the running charm quark mass from

the D and D_s mesons. The result in Eq. (10) confirms earlier results obtained to two- and three-loop accuracies [8,2].

• Studied the mass-splittings of the 0^+-0^- in the D systems using QSSR. Our result in the (0^+) channel given in Eq. (11) agrees with the recent experimental findings of the $D_{sJ}(2317)$ suggesting that this state is a good candidate for being a $\bar{c}s$ 0^+ meson.

• Found, in Eq. (13), that the $SU(3)$ breaking responsible of the mass-splitting between the $D_s(0^+)$ and $D(0^-)$ is small of about 25 MeV contrary to the case of the pseudoscalar D_s-D mesons of about 100 MeV.

• Extended our analysis to the B-system. Our results in Eqs. (11), (15) and (1) suggest an approximate (light and heavy) flavour and spin symmetries of the meson mass-splittings. We use this result to get the mass of the $\bar{c}s$ $D_s^*(1^+)$ meson in Eq. (17), which is in (surprising) good agreement with the observed $D_{sJ}^*(2457)$.

• Also determined the decay constants of the 0^+ mesons and compare them with the ones of the 0^- states. The result in Eq. (18), which is similar to the pseudoscalar decay constant $f_D \simeq 205$ MeV, suggests a huge violation of the heavy quark symmetry $1/\sqrt{M_D}$ scaling law. Finally, our results in Eqs. (19) and (20) indicate that the $SU(3)$ breaking act in an opposite way compared to the case of the 0^- channels.

We expect that experimental measurements will test the validity of the results obtained to two-loop accuracy in this paper from QCD spectral sum rules. However, a complete confirmation of the nature of these new states needs a detail study of their production and decays, which we plan to do in the future. We also expect that these results will be an useful guideline for the lattice QCD calculations like were the case of various sum rule results in the past.

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